Short Papers

Extended Cavity Perturbation Technique to Determine the Complex Permittivity of Dielectric Materials

Binshen Meng, John Booske, and Reid Cooper

Abstract—**An improved measurement technique to determine the complex dielectric properties of materials has been developed that extends the validity of the conventional cavity perturbation technique for circular cylindrical rod-shaped samples in circular cylindrical cavities resonating in TM_{0n0} modes. The method is particularly useful for the dielectric characterization of fragile, low-loss materials that are difficult to machine to typically required thin dimensions. The method further allows for multi-frequency measurements using higher-order radial modes and somewhat alleviates the very small cavity dimensions typically required by the conventional perturbation technique at higher microwave frequencies. A validity criterion for the extended method is given. Measurements of the complex permittivity of NaCl single crystals are presented, showing excellent agreement with theory.**

I. Introduction

A variety of methods of measuring dielectric constants have been developed in the last several decades [1]. Transmission line methods have been found to be appropriate for lossy materials. Free space methods have been used successfully to characterize low-loss materials over a broad frequency range, but they are sometimes difficult to implement accurately since they involve a host of special problems, such as the suppression of unwanted (multiple) reflections, the launching of a plane wave in a limited space, diffraction from the edges of the sample, and the need for very large, uniform sheets of material (especially at low frequencies).

The cavity perturbation technique has been extensively and successfully employed to measure the complex dielectric constants of low-loss materials [2], [3]. These measurements are performed by inserting a small sample with a certain shape into a microwave resonant cavity and determining the real part and the imaginary part of the complex permittivity from the shift of the resonance frequency and the change of the cavity Q factor, respectively.

The conventional cavity perturbation method may be difficult to perform in some practical situations, however, due to the requirement that the sample volume must be very small to produce a negligible perturbation to the electromagnetic field distribution inside the cavity. For the case of a circular cylindrical cavity operating in a TM_{0n0} mode, the sample is usually a thin circular cylindrical rod. In some applications, the sample materials may be fragile and extremely difficult to fashion into rods thin enough to satisfy the accuracy requirements for higher frequencies or higher order modes. Considerable errors may result from the use of the conventional cavity perturbation technique with thicker rods.

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In this paper, we describe an accurate method to determine the complex dielectric constant of rod-shaped dielectric materials with larger diameters from the change of the resonant frequency and the Q factor for circular cylindrical cavities resonating in TM_{0n0} modes.

II. Measurement Theory

A. Review of the Conventional Cavity Perturbation Theory

The change of a complex eigenfrequency $\tilde{f}$ caused by a small sample having volume V_s with the complex dielectric constant ϵ and permeability μ in a cavity with volume V_c is [4]

$$\frac{\Delta\tilde{f}}{f_0} = -\frac{\int_{V_s}(\Delta\epsilon\,\mathbf{E}\cdot\mathbf{E}_0^* + \Delta\mu\mathbf{H}\cdot\mathbf{H}_0^*)d\tau}{\int_{V_c}(\epsilon\,\mathbf{E}\cdot\mathbf{E}_0^* + \mu\mathbf{H}\cdot\mathbf{H}_0^*)d\tau} \tag{1}$$

where $\mathbf{E}_0$ and $\mathbf{H}_0$ represent electric and magnetic fields respectively in the empty cavity, and $\mathbf{E}$ and $\mathbf{H}$ represent the corresponding quantities in the cavity with the small sample. For the purposes of further discussion, we will restrict our attention to materials for which the permeability μ is a constant, hence $\Delta\mu = 0$. When the sample is small, it is reasonable in the empty region of the sample-loaded cavity to approximate $\mathbf{E}$ and $\mathbf{H}$ by $\mathbf{E}_0$ and $\mathbf{H}_0$. Denoting the value of $\mathbf{E}$ inside the sample by $\mathbf{E}_{\mathrm{int}}$, we have [5]

$$\frac{\Delta\tilde{f}}{f_0} \simeq -\frac{\int_{V_s}\Delta\epsilon\,\mathbf{E}_{\mathrm{int}}\cdot E_0^* d\tau}{2\int_{V_c}\epsilon|E_0|^2 d\tau}. \tag{2}$$

The change of the complex eigenfrequency can be related to the changes in the resonance frequency $f = \mathrm{Re}(\tilde{f})$ and the Q factor of the cavity, through the relation [6]

$$\frac{\Delta\tilde{f}}{f_0} = \frac{\Delta f}{f_0} + j\frac{1}{2}\left(\frac{1}{Q} - \frac{1}{Q_0}\right) \tag{3}$$

where Q and Q_0 are the quality factors of the cavity with and without the sample, respectively.

Generally the complex dielectric constant ϵ can be written as

$$\epsilon = \epsilon_0(\epsilon' - j\epsilon'') \tag{4}$$

so that

$$\begin{aligned}\frac{\Delta\tilde{f}}{f_0} &\simeq -\frac{\int_{V_s}(\epsilon' - 1 - j\epsilon'')\mathbf{E}_{\mathrm{int}}\cdot E_0^* d\tau}{2\int_{V_c}|E_0|^2 d\tau}\\ &= -\frac{\int_{V_s}(\epsilon' - 1)\mathbf{E}_{\mathrm{int}}\cdot E_0^* d\tau}{2\int_{V_c}|E_0|^2 d\tau} + j\frac{\int_{V_s}\epsilon''\mathbf{E}_{\mathrm{int}}\cdot E_0^* d\tau}{2\int_{V_c}|E_0|^2 d\tau}.\end{aligned} \tag{5}$$

Comparing (5) to (3) yields

$$\epsilon' \simeq 1 - 2C_{\mathrm{conv}}\frac{\Delta f}{f_0} \tag{6}$$

and

$$\epsilon'' \simeq C_{\mathrm{conv}}\left(\frac{1}{Q} - \frac{1}{Q_0}\right) \tag{7}$$

where

$$C_{\mathrm{conv}} = \frac{\int_{V_c}|E_0|^2 d\tau}{\int_{V_s}\mathbf{E}_{\mathrm{int}}\cdot E_0^* d\tau}. \tag{8}$$

For a circular cylindrical cavity resonating in TM_{0n0} modes with a cavity radius of a, a cylindrical rod-shaped sample radius of b, and a cavity height of d, use of the quasistatic approximation leads to [5]

$$C_{\text{conv}} = \frac{\int_0^a J_0^2(k_n r) r\,dr}{\int_0^b J_0^2(k_n r) r\,dr} \tag{9}$$

where $k_n = 2\pi f_n \sqrt{\epsilon_0 \mu_0}$, $f_n = c x_{0n}/2\pi a$, and x_{0n} is the nth zero of the Bessel function J_0.

Equations (6) and (7) are the basic expressions on which conventional cavity perturbation measurement methods are based. The following is a list of the assumptions made in the derivation of the two equations:

1) $\mathbf{E} \approx \mathbf{E}_0$ in the denominator of (2) (this is equivalent to the physical assumption that the stored energy in the empty cavity equals that in the cavity with the sample).
2) Use of the quasistatic approximation $\mathbf{E}_{\text{int}}(\mathbf{r}) \approx \mathbf{E}_0(\mathbf{r})$ to calculate C_{conv} in (9) for a circular cylindrical cavity resonating in TM_{0n0} modes.
3) In many applications, the calculation of C_{conv} has been further restricted in validity by the additional assumption that $\mathbf{E}_{\text{int}}(\mathbf{r}) \approx \mathbf{E}_0(\mathbf{r}) \approx \mathbf{E}_0$ takes a constant value throughout the sample.
4) Consistent with assumption #1, one assumes that the difference between the quality factors of the cavity due to the cavity wall loss with and without the sample is negligible. Note that assumption #1 implies assumption #4, but not the converse.

B. Extended Measurement Theory

This method involves solving the eigenvalue problem of a dielectric sample in a resonant cavity. For the circular cylindrical cavity metioned earlier, solving Maxwell's equations with boundary conditions we obtain the dispersion relation

$$\frac{k_0}{k}\frac{J_0(kb)}{J_1(kb)} = \frac{J_0(k_0 b)N_0(k_0 a) - J_0(k_0 a)N_0(k_0 b)}{J_1(k_0 b)N_0(k_0 a) - J_0(k_0 a)N_1(k_0 b)}. \tag{10}$$

For low-loss materials, this equation can be solved for the resonant frequencies by an approximation of $\epsilon \approx \text{Re}(\epsilon) = \epsilon'$. Equation (10) can be solved for arbitrary complex ϵ. We then obtain the eigenmode fields

$$E_{z1} = AJ_0(k_n r) \quad \text{for } r \le b \tag{11}$$

and

$$E_{z2} = AJ_0(k_n b)\frac{J_0(k_{0n}a)N_0(k_{0n}r) - N_0(k_{0n}a)J_0(k_{0n}r)}{J_0(k_{0n}a)N_0(k_{0n}) - N_0(k_{0n}a)J_0(k_{0n}b)} \quad \text{for } b < r \le a \tag{12}$$

where $k_n^2 = \omega_n^2 \mu_0 \epsilon_0 \epsilon'$ and $k_{0n}^2 = \omega_n^2 \mu_0 \epsilon_0$. The stored energy is given by

$$\begin{aligned} W &= \frac{\epsilon}{2}\int |E_z|^2 d\tau \\ &= \frac{\epsilon_0}{2}\left(\epsilon' \int_{r<b} |E_{z1}|^2 d\tau + \int_{b \le r \le a} |E_{z2}|^2 d\tau\right). \end{aligned} \tag{13}$$

The power dissipation P_w due to the conduction on the surface of the cavity wall can be calculated in the conventional manner as the surface integral (over cavity walls) of $|H_t|^2 R_s/2$, where R_s is the surface resistance and $H_t = H_\phi = -(j\omega\epsilon/k^2)dE_z/dr$ is the component of the magnetic field tangential to the surface of the cavity wall. H_t can be related to the electric field as

$$H_t = H_\phi = -\frac{j\omega\epsilon}{k^2}\frac{dE_z}{dr}. \tag{14}$$

We then obtain the quality factor considering the loss on the cavity wall and the refractive effect of the sample on the field distribution but neglecting the effects of sample losses

$$Q_w = \omega \frac{W}{P_w}. \tag{15}$$

Since the power dissipation due to the sample is

$$P_s = \int_{r<b} \frac{1}{2}\omega\epsilon_0\epsilon''|E_z|^2 d\tau \tag{16}$$

the total Q factor (with the sample in the cavity) will be

$$\frac{1}{Q} = \frac{1}{Q_w} + \frac{P_s}{\omega W}. \tag{17}$$

We next assume that the difference between Q_w and Q_0 (quality factor of the cavity without samples) is negligible (we discuss the impact of this assumption at a later point in the discussion). Then, with the help of (16) we have

$$\epsilon'' = C_{\text{exact}}\left(\frac{1}{Q} - \frac{1}{Q_w}\right) \approx C_{\text{exact}}\left(\frac{1}{Q} - \frac{1}{Q_0}\right) \tag{18}$$

where

$$C_{\text{exact}} = \frac{\epsilon' \int_{r<b} |E_{z1}|^2 d\tau + \int_{b \le r \le a} |E_{z2}|^2 d\tau}{\int_{r<b} |E_z|^2 d\tau}. \tag{19}$$

The constant C_{exact} is considered to represent an "exact" calculation as it employs the general field distribution solutions of (11) and (12), whose only *a priori* assumption was that the cavity walls were made of a good conductor (i.e., $\frac{\sigma}{\omega\epsilon_0} \gg 1$) material. The quality factor of the empty cavity [7] can be calculated as

$$Q_0 = \frac{1}{2}\sqrt{\frac{\sigma}{\pi f_{0n0}\epsilon_0}}\frac{x_{0n}}{a/d + 1} \tag{20}$$

where σ is the conductivity of the cavity wall material, x_{0n} is the nth zero of the Bessel function J_0, and f_{0n0} is the resonance frequency of the TM_{0n0} mode.

To illustrate the advantages of the extended method, we describe its application to a specific experimental system and compare its accuracy with that of the conventional method. The experimental configuration employs a circular cylindrical cavity operating in the first three TM_{0n0} ($n \le 3$) modes. With an inner radius of $a =$ 5.22 cm and a height of 2 mm, the resonant frequencies of these three eigenmodes are 2.20, 5.05, and 7.91 GHz. A cylindrical sample rod is inserted through a small hole on one of two flat walls. For current research purposes, we have been investigating microwave absorption mechanisms in NaCl single crystals. Therefore, our interest in these higher order modes is based on a desire to characterize the sample's complex permittivity at several different frequencies without removing the sample from the cavity. However, the brittle ceramic properties of NaCl make it difficult to obtain thin single crystal rods with a sample radius less than 2 mm. The combination of these interests and constraints necessitated the development of the extended cavity perturbation method.

Comparing the predictions for ϵ' from resonant frequency shift by using (10) for the extended method with those by (6) for the conventional approach (dashed curve) we found that their difference for the lowest order mode (TM_{010}) is probably negligible but that the conventional determination of ϵ' errs by 10% for the TM_{020} mode and more than 20% for the TM_{030} mode.

TABLE I
SOME CALCULATED PARAMETERS AT THE DIFFERENT RESONANT FREQUENCIES

$f_{0n0}\,(GHz)$	Q_0	Q_w	C_{conv}	C_{exact}
2.1985	10435	10444	183.66	175.77
5.0465	15808	15859	79.61	62.29
7.9113	19794	19872	51.50	35.24

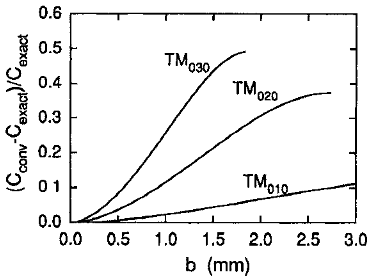

Fig. 1. The relative difference between constant C for the conventional perturbation technique and extended methods versus the radius of the sample.

Only one basic assumption has been made in deriving (18) and (19) for ϵ'', i.e., that the measured empty cavity quality factor Q_0 is to a good approximation equivalent to the quality factor Q_w associated with cavity wall losses in the presence of the sample (but ignoring the sample losses). This assumption is also made in the conventional cavity perturbation method.

The quality factors Q_0 and Q_w have been calculated from (20) and (15), respectively, and tabulated in Table I for the first three TM_{0n0} modes in our experimental cavity (described above) and (in the case of Q_w) for a 2 mm radius NaCl rod with an estimated permittivity (real part) of $\epsilon' \approx 5.6$ (based on literature values [8]). The difference between Q_0 and Q_w is very small (less than 0.5% which is smaller than the measurement error of 1%). Hence, an accurate determination of Q_w can be obtained by measuring Q_0 in the empty cavity. This is very important because one can never actually measure Q_w directly. Table I also includes calculated values for C_{conv} and C_{exact} for the same three eigenmodes. It is apparent that significant errors can be incurred in obtaining ϵ'' from the conventional calculation due to the significant error in C_{conv} as compared to C_{exact}.

The limitations of the conventional method for larger sample radii and higher order modes is further illustrated in Fig. 1. In this figure we have plotted the relative error $(C_{\text{conv}} - C_{\text{exact}})/C_{\text{exact}}$ in the conventional calculation of the integrated-field-distribution constant as a function of sample radius b in our cavity (again, we assume $\epsilon' = 5.6$ for this illustration). For the lowest order mode, the conventional method provides reasonable accuracy (less than 10% error) for a sample radius up to approximately 3 mm. However, similar accuracy for the second and third order modes would require sample radii less than 1.0 and 0.5 mm, respectively. These small dimensions are prohibitively difficult to achieve with single crystal specimens of most (brittle) ionic solids.

As mentioned above, the assumption $Q_0 \approx Q_w$ does not impose significant errors on the determination of ϵ''. The absolute error resulting from this assumption can be calculated as

$$\Delta\epsilon'' = C_{\text{exact}}\left(\frac{1}{Q} - \frac{1}{Q_w}\right) - C_{\text{exact}}\left(\frac{1}{Q} - \frac{1}{Q_0}\right) = C_{\text{exact}}\left(\frac{1}{Q_0} - \frac{1}{Q_w}\right). \quad (21)$$

For the cavity dimensions mentioned earlier, this error is less than 1.5×10^{-5} for each mode with a sample radius of $b = 2$ mm and is still very small ($<2 \times 10^{-5}$) for b up to 3 mm. This amount of error is much less than the minimum measurable value of $\epsilon'' \sim 10^{-4}$ determined by the precision of our experimental measurement system. Therefore, the assumption that $Q_0 \approx Q_w$, is accurate for a fairly wide (and practical) range of sample radii.

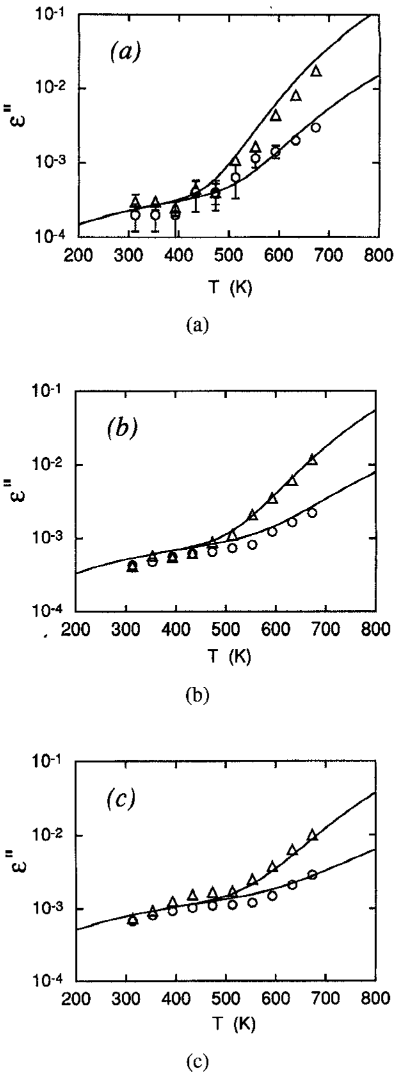

Fig. 2. The experimentally measured ϵ'' and theoretically predicted ϵ'' in NaCl single crystals at the frequencies of (a) 2.2 GHz, (b) 4.8 GHz, and (c) 7.4 GHz versus temperature. In all of the plots, o: undoped crystal ("pure" crystal), △: doped crystal, —: theoretical prediction.

We now take into account the frequency pulling effect of the sample insertion hole. The frequency pulling affects Q measurements and the determination of ϵ' (thus C_{exact}) so that it introduces error to ϵ''. Employing the correction method developed by Estin [9], the corrections of ϵ'' for the TM_{0n0} ($n = 1, 2$, and 3) modes are 0.12%, 0.44%, and 0.27% for sample radius $b = 2$ mm, and 0.19%, 0.15%, and 1.8% for $b = 3$ mm. All of these corrections are much less than the measurement error of 1%, except the last one which is still comparable to the measurement error.

III. EXPERIMENTAL RESULTS

Experiments have been performed in our system using the extended measurement theory. In particular, we have measured ϵ'' in a nearly-pure (i.e., undoped) commercial NaCl single crystal sample and a second single crystal specimen doped with approximately 250 ppm Ca^{++} ion impurity concentration. The measurements were performed over the temperature range of 300–700°K for the TM_{010}, TM_{020}, and TM_{030} resonances. The values of the three resonant frequencies—2.2, 4.8, and 7.4 GHz-are shifted from the empty cavity values due to the presence of the sample. Based on the extended method, we find that the frequency shift data at all three frequencies yield a value of $\epsilon' \approx 5.64 \pm 0.05$. This result is also in very close agreement with previous measurements [8].

Equations (18) and (19) have been used to obtain ϵ'' from experimentally measured Q values with and without the sample. The results are plotted in Fig. 2 along with theoretical predictions [10] based on a model for microwave absorption that includes ionic conduction [8], defect-complex-dipole relaxation [8] and multi-phonon quasiresonance [11] processes. The results of the experimental measurements and the model predictions are in excellent, consistent agreement over a very large temperature range and for a significant range of frequencies.

IV. Conclusion

The measurement method described in this paper extends the validity of the cavity perturbation technique to larger samples and multiple cavity modes, provided that the difference between Q_w and Q_0 is still negligible in comparison with the measurement precision. This extended applicability is advantageous for determining microwave dielectric properties of many important ionic crystalline solids that are difficult to fabricate into very thin rods. As an illustration, the method has been successfully used to study the dielectric properties of NaCl crystals in the microwave frequency regime.

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Characterization of Microstrip Discontinuities Using Conformal Mapping and the Finite-Difference Time-Domain Method

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Abstract— **Microstrip discontinuities are analyzed using Wheeler's waveguide model and the finite-difference time-domain (FDTD) method. Wheeler's model employs a conformal transformation to convert a microstrip into an enclosed waveguide structure. This permits the mapping of a discontinuous microstrip into a discontinuous, but enclosed, waveguide. The enclosed waveguide eliminates the difficulties usually associated with analysis of an open domain geometry. The FDTD technique is then used to calculate the scattering coefficients of the discontinuous waveguide. The features of this approach are: 1) it yields a smaller computational domain than that required to analyze the untransformed geometry; 2) it yields results over a band of frequencies; and 3) it is simple to implement. Results obtained using this scheme show good agreement with previously published results.**

I. Introduction

Many modern microwave and millimeter-wave integrated circuits guide the transmission of energy using microstrip lines (or asymmetrical striplines). The passive components in these circuits are often constructed from microstrip discontinuities. To analyze and synthesize microwave integrated circuits, it is essential to accurately model the frequency-dependent properties of these discontinuities. The frequency-dependent properties of microstrip lines, in the absence of discontinuities, can be obtained from simple empirical formulae that accurately describe the phase velocities and the characteristic impedances of the fundamental and higher-order modes [1]. In the presence of discontinuities, analysis becomes quite cumbersome and several solution techniques have been proposed. These techniques are based on any one of a number of methods including mode matching [2]–[7], finite-difference time-domain (FDTD) [8]–[11], method of moments (MoM) [12]–[16], finite element method (FEM) [17], [18], and the measured equation of invariance (MEI) [19].

All of the aforementioned techniques have inherent limitations. For example, solutions based on mode matching can become unwieldy for even slightly complicated geometries. MoM, FEM, and MEI solutions can be expensive when results are desired over a broad spectrum. Direct application of FDTD to these circuits can require the use of a large and/or fine mesh which, in turn, requires long computation times and large amounts of computer memory.

This paper presents a technique that is both simple to implement and computationally inexpensive. The technique works by converting the open microstrip structure into an enclosed waveguide using the conformal mapping technique described by Wheeler [20], [21] and then using the conventional FDTD technique to analyze the discontinuities. The conformal mapping reduces the problem to one with no stray fields which greatly reduces the size of the computational domain. Since this is a time-domain technique, results can be obtained over a band of frequencies via Fourier transforms. However, since the conformal mapping is only accurate at lower frequencies, the cost of using this simplified approach is that the

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